Overview of Harmonic Balance FEM and its Application in Harmonic Domain Nonlinear EM Field

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The state of the art of harmonic balance finite element method (HBFEM) is to use harmonic balance theories and finite element method based computational electromagnetics (CEMs) technology to analyse or investigate nonlinear electromagnetic fields with harmonic problems in electrical and electronic engineering. HBFEM can directly solve the steady-state response of the electromagnetic field in the multi-frequency domain. The method is often considerably more efficient and accurate in capturing coupled nonlinear effects than the traditional FEM time-domain approach when the field exhibits widely separated harmonics in the frequency spectrum domain and mild nonlinear behavior. This paper presents an overview of HBFEM and its application in solving various harmonic problems related to high frequency transformers, DC biased transformers, and geomagnetic induced currents on transformers.

***Index Terms*— Computational electromagnetics (CEMs), Harmonic balance finite element method (HBFEM), Multi-frequency domain (MFD).**

I. Introduction

THE HARMONIC balance technique was first introduced to analyse low frequency electromagnetic (EM) field problems in the late 1980s [1]. It was followed by the contributions of Lu in various applications [2-3], and other researchers [4-6]. Harmonic balance techniques were combined with the finite element method (FEM) to accurately solve the problems arising from time-periodic steady-state nonlinear magnetic fields. The method can be used for weak and strong nonlinear time periodic EM fields (including DC biased transformers and geomagnetic induced currents on transformer problems), as well as harmonic problems in renewable energy systems with distributed energy resources.

II. The Basic Concept Of HBFEM

The harmonic balance FEM (HBFEM) method uses a linear combination of sinusoids to build the solution, and represents waveforms using the sinusoid; coefficients combined with the finite element method. It can directly solve the steady-state response of the EM field in the multi-frequency domain. Thus, the method is often considerably more efficient and accurate in capturing coupled nonlinear effects than the traditional FEM time-domain approach when the field exhibits widely separated harmonics in the frequency spectrum domain, e.g. Pulse Width Modulation (PWM) case. The harmonic balance FEM consists of approximating the time periodic solution (magnetic potentials, currents, voltages, etc.) with a truncated Fourier series. Besides the frequency components of the excitation (e.g. applied voltages), the solution contains harmonics due to nonlinearity (magnetic saturation and nonlinear lumped electrical components) and movement (e.g. rotation). The HBFEM leads to a very large, single system of algebraic equations. Dependent on the problem at hand, it may be much more efficient than the time domain approach (time stepping). Indeed, the latter inevitably requires stepping through the transient phenomenon before reaching the quasi-steady-state. The global HBFEM system of algebraic equations is derived in an original way. The Galerkin approach is applied to both the space and time discretization.The time harmonic basis functions are used for approximating the periodic time variation as well as for weighing the time domain equations in the fundamental period. Magnetic saturation and nonlinear electrical circuit coupling are thus easily accounted for by means of the Newton-Raphson method. Rotation in FEM models of rotating machines, using the

TABLE I
COMPARISON OF NUMERICAL COMPUTATION OF TIME-PERIODIC STEADY-STATE NONLINEAR ELECTROMAGNETIC FIELD

	Frequency domain method	Step-by-step method	Time-periodic method	Harmonic balance method
Computation domain	Single frequency domain	Time domain	Time domain	Multiple frequency domain
Nonlinear and harmonic problems	For weak nonlinear fields, but not for harmonic problems	For weak nonlinear fields and harmonic problems	For weak nonlinear fields and harmonic problems	For weak and strongly nonlinear fields and harmonic problems
Widely separated harmonics	Cannot compute	Difficult to compute	Difficult to compute	Easy to compute (e.g. PWM)
Computation time depending on	Number of degrees of freedom	Time step and number of degrees of freedom	Time step and number of degrees of freedom	Harmonic number and number of degrees of freedom
Computation accuracy	Large error at high frequency harmonics	Truncation error	Truncation error	Number of harmonics considered
Calculation of harmonic components	Impossible	Calculate from computa-tion results	Calculate from computa-tion results	Calculate several harmonics simulta-neously
Post-processing of harmonics	Impossible	Indirectly	Indirectly	Directly obtained

moving boundary technique, can be considered as well. The HBFEM has been validated by applying it to several test cases (transformer feeding a rectifier bridge, various synchronous and asynchronous machines, DC biased transformer, etc.). The harmonic waveforms of the magnetic field, currents and voltages etc., are shown to converge well compared to those obtained with time stepping as the spectrum of the HBFEM analysis is extended. The comparison results between HBFEM and other numerical methods are illustrated in Table I.

III. Harmonic Balance FEM in EM Fields

Harmonic balance can be applied to EM field analysis as the fields that contain the harmonics also satisfy Maxwell's equations. The harmonics generated in EM fields can be described in the following three ways:

- When a linear EM object is excited by sources which contain the harmonics, it will exhibit the harmonic field.
- When a nonlinear EM object is excited by a sinusoidal signal, it will exhibit harmonic fields.
- When both linear and nonlinear EM objects are excited by the sources which contain the harmonics, the result is a complex harmonic field.

One of the most obvious properties of a nonlinear system is the generation of harmonics. For example, we use the following equations to describe the quasi-static EM fields. These can be defined as follows:

A. Nonlinear Electromagnetic field

Nonlinear Magnetic field:

$$\nabla \times \nu \nabla \times A + \sigma(\partial A / \partial t + \nabla \varphi) - J_s = 0 \quad (1)$$

Nonlinear electric field:

$$\nabla \cdot \{\sigma E + \varepsilon(\partial E / \partial t)\} = 0 \quad (2)$$

where the electric field ***E***, magnetic vector potential ***A***, scalar potential $\boldsymbol{\varphi}$ on the arbitrary node ***i*** in the discretised system, the electrical conductivity σ, dielectric permittivity $\boldsymbol{\varepsilon}$ and the source current density ***J***s can be respectively expressed as:

$$A^i = A_0^i + \sum_{k=1}^{\infty}\{A_{ks}^i \sin(k\omega t) + A_{kc}^i \cos(k\omega t)\} \quad (3)$$

$$\varphi^i = \varphi_0^i + \sum_{k=1}^{\infty}\{\varphi_{ks}^i \sin(k\omega t) + \varphi_{kc}^i \cos(k\omega t)\} \quad (4)$$

$$E^i = E_0^i + \sum_{k=1}^{\infty}\{E_{ks}^i \sin(k\omega t) + E_{kc}^i \cos(k\omega t)\} \quad (5)$$

$$J_s = J_0 + \sum_{k=1}^{\infty}\{J_{ks} \sin(k\omega t) + J_{kc} \cos(k\omega t)\} \quad (6)$$

where the vector $\boldsymbol{A_0}$, $\boldsymbol{E}_0$, $\boldsymbol{J}_0$ and scalar φ_0 are the DC components respectively, and *ks* and *kc* represent the *sin* and *cos* components. In practical applications, harmonic *k* is not infinite. Only a finite number is required in the real system.

B. Nonlinear Medium Description

Nonlinear phenomena in EM fields are caused by nonlinear materials. The nonlinear materials are normally field strength dependent. Therefore, when the time-periodic quasi-static EM field is applied to the nonlinear material, the electromagnetic properties of the material will be functions of the EM field. They will also be time dependent.

The magnetic reluctivity $\boldsymbol{\nu}$ corresponding to ***B***(t) can be expressed as:

$$\nu(t) = H(B(t)) / B(t) = \nu_0 + \sum_{k=2n-2}^{\infty}\{\nu_{ks} \sin(k\omega t) + \nu_{kc} \cos(k\omega t)\} \quad (7)$$

The electrical conductivity σ related E(t) can be expressed as :

$$\sigma(t) = \sigma(E(t)) = \sigma_0 + \sum_{k=2n-2}^{\infty}\{\sigma_{ks} \sin(k\omega t) + \sigma_{kc} \cos(k\omega t)\} \quad (8)$$

C. Boundary Conditions

Since the trigonometric functions are orthogonal functions, the harmonic potential $\boldsymbol{P}_k$ (degrees of freedom) on the boundary satisfy Dirichlet and Neumann boundary conditions. The frequency-domain representation, or spectrum on each boundary node, can then be expressed as follows:

Dirichlet boundary condition:

$$P_k = \{P_0,\ P_{1s},\ P_{1c},\ P_{2s},\ P_{2c},\ \cdots P_{ks},\ \ P_{kc}\}^T \quad (9)$$

Neumann boundary condition:

$$\frac{\partial P_k}{\partial n} = \{\frac{\partial P_0}{\partial n},\ \frac{\partial P_{1s}}{\partial n},\ \frac{\partial P_{1c}}{\partial n},\ \frac{\partial P_{2s}}{\partial n},\ \frac{\partial P_{2c}}{\partial n},\ \cdots \frac{\partial P_{ks}}{\partial n},\ \ \frac{\partial P_{kc}}{\partial n}\}^T \quad (10)$$

where the *k* is the harmonic number, potential $\boldsymbol{P_{ks}}$ and $\boldsymbol{P_{kc}}$ are the sum of the harmonics on each boundary node *i*.

D. The Generalized HBFEM

The system matrix equation for current source excitation can then be written in a compact form:

$$[S]\{A\} + [M]\{A\} - \{K\} = 0 \quad (11)$$

where [S] is the system matrix and [M] is the harmonic related matrix, and {*K*} is related to excitation source. All harmonic components of magnetic vector potential ***A*** can be directly obtained by solving this system matrix equation.

The detailed discussion and new application in DC biased transformers and geomagnetic induced currents on transformer problems will be presented in the full paper.

IV. References

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